

Express Mail No. EV442675628 US
Application No. 09/665,179
Title: Method & Apparatus for Determining
Colimits of Hereditary Diagrams
Inventors: Dusko Pavlovic, et al.
Atty. Docket No.: 11128-04483

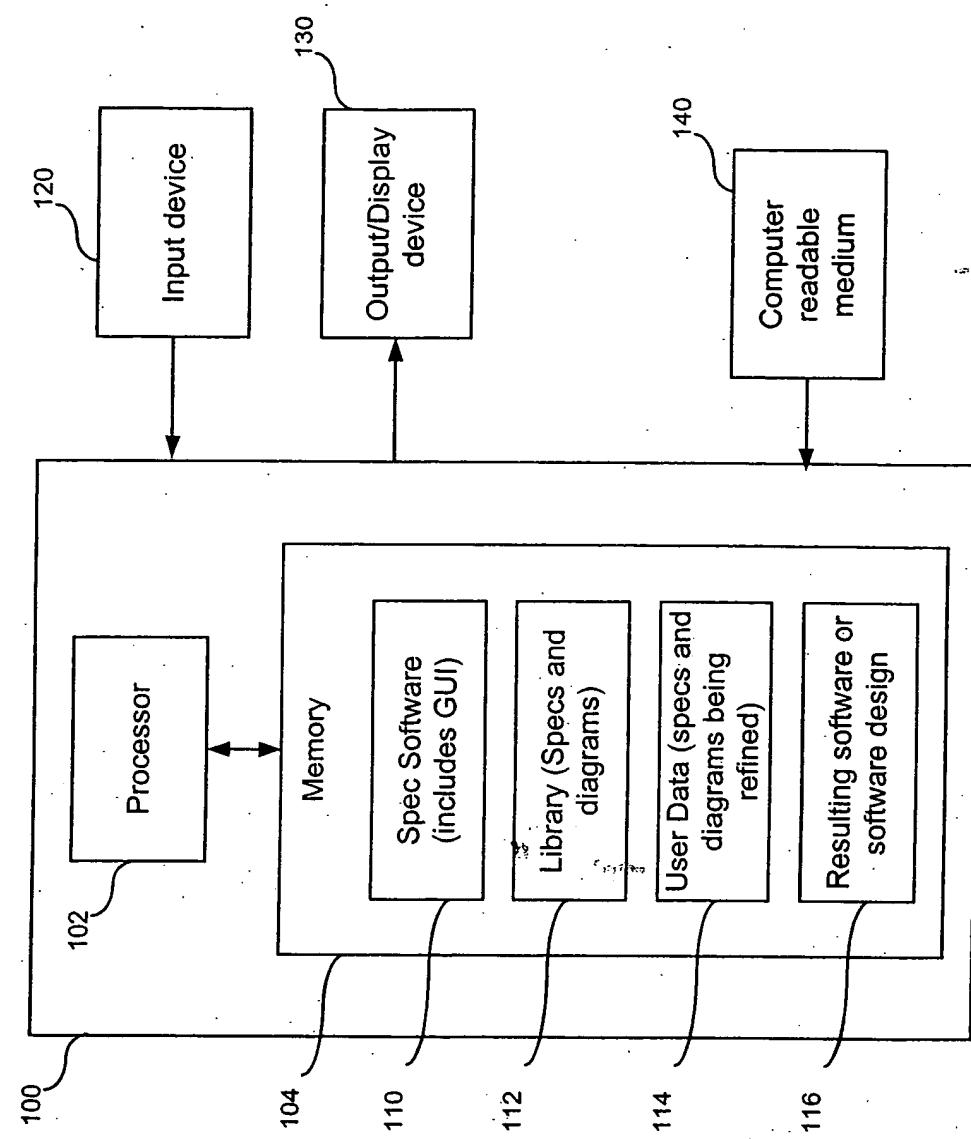


Fig. 1



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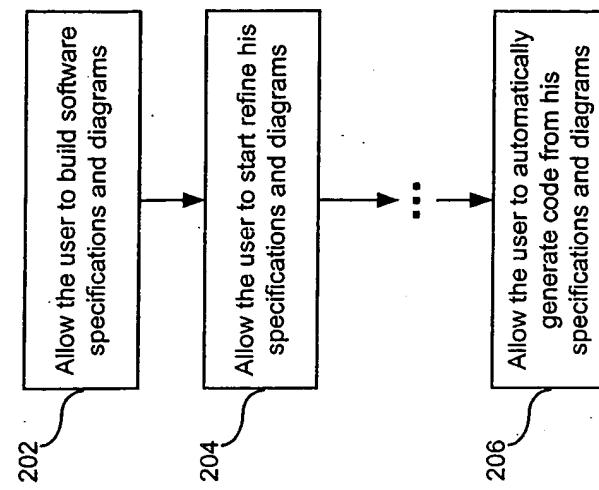
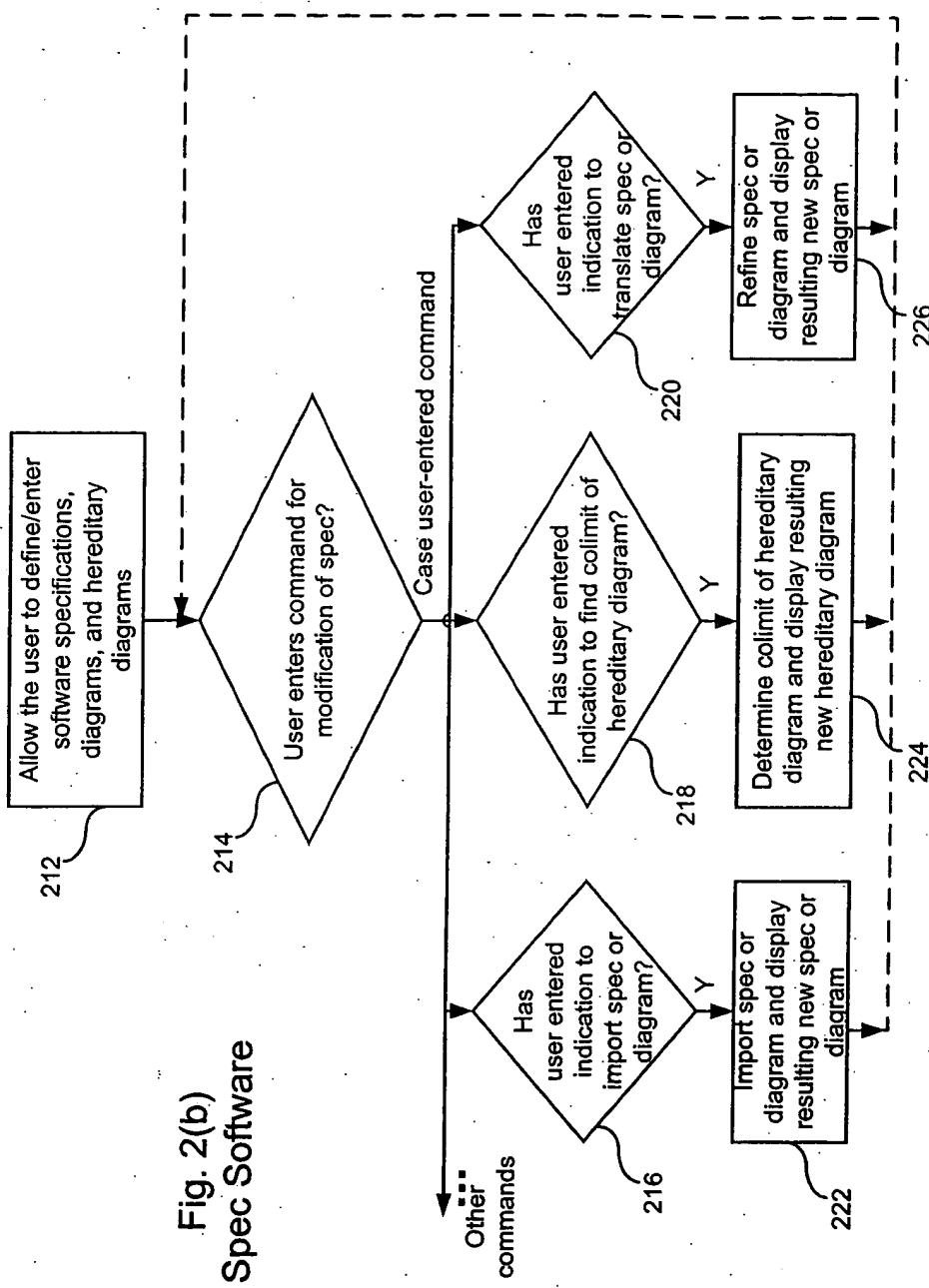
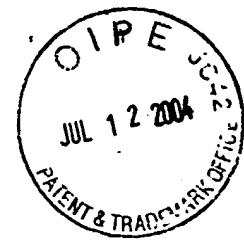
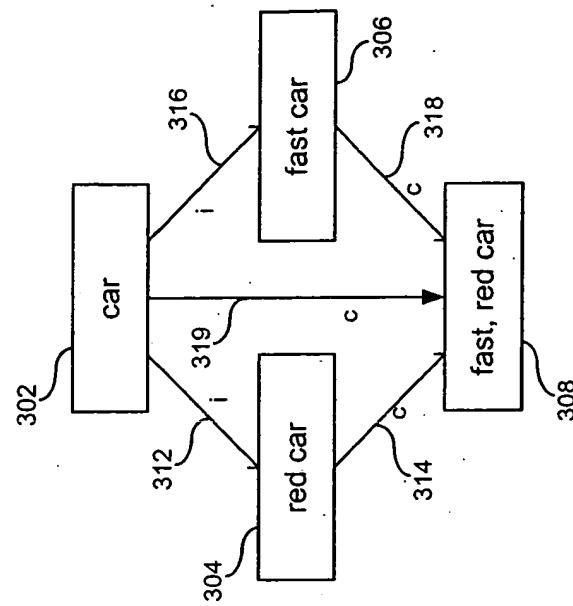


Fig. 2(a)
Spec Software

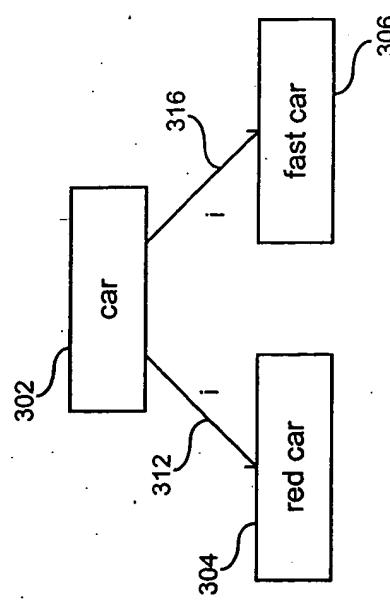




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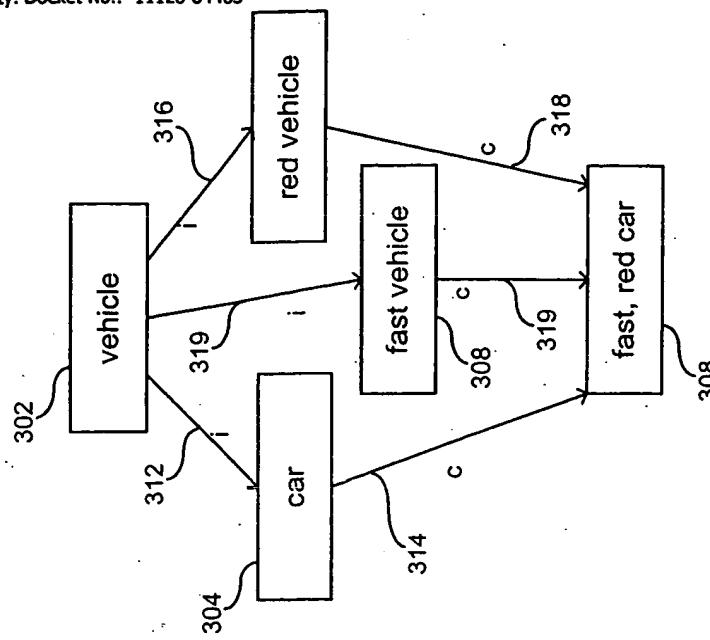
Refining a Specification
Fig. 3(a)



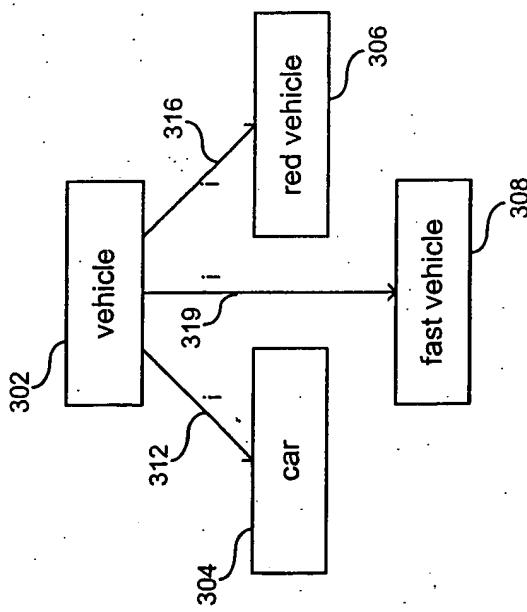
Example of Using a Colimit to
Combine Refined Specifications
Fig. 3(b)



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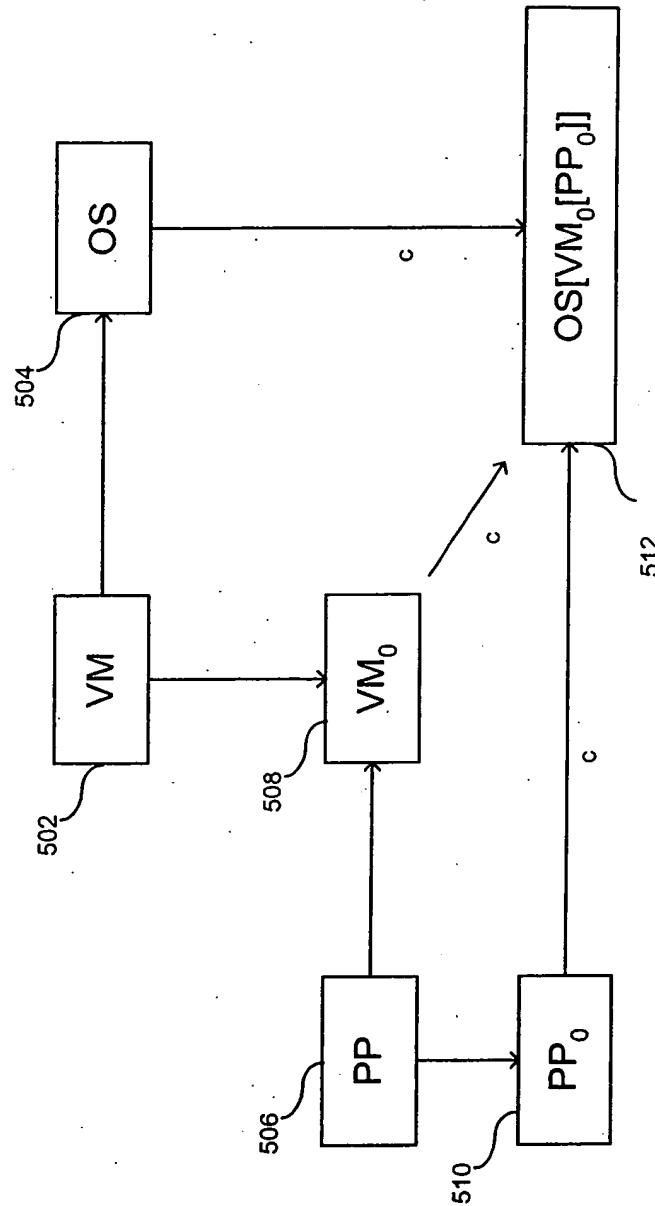
Example of Using a Colimit to
 Combine Refined Specifications
 Fig. 4(b)



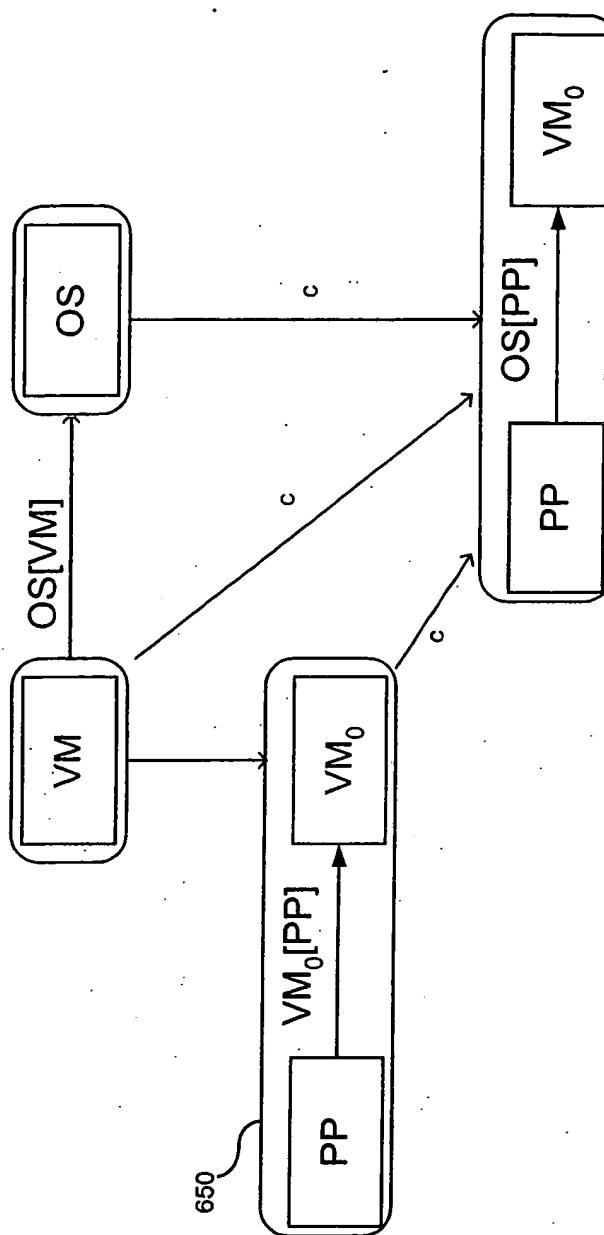
Refining a Specification
 Fig. 4(a)



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Example Colimit of Specifications
Fig. 5



Example Colimit of Diagrams of Diagrams

Fig. 6



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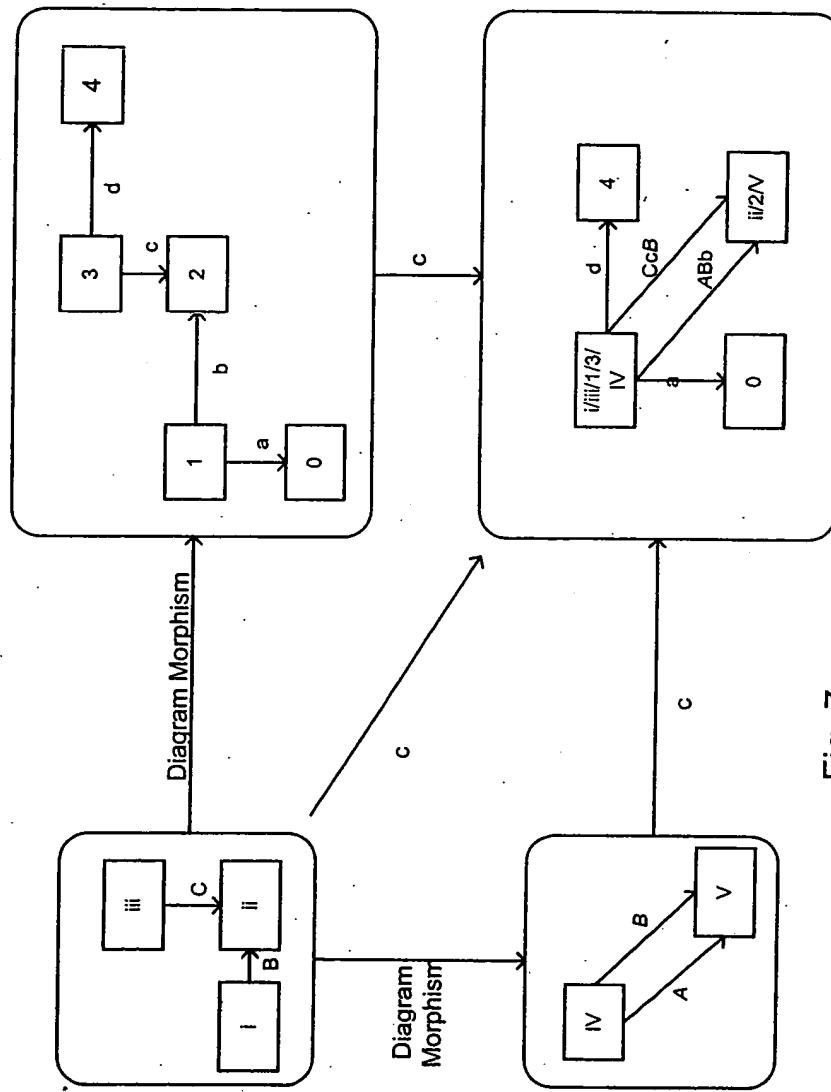
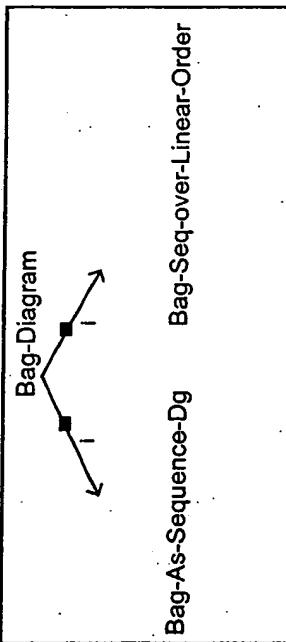
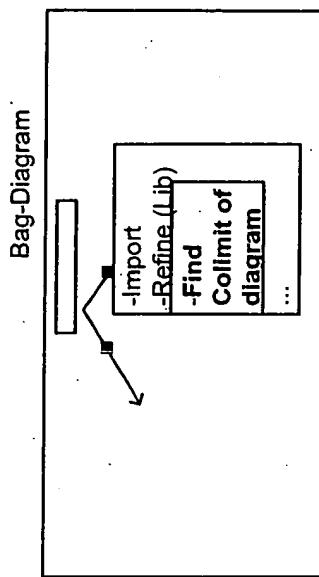


Fig. 7
Example of Taking the Colimit of Hereditary Diagrams

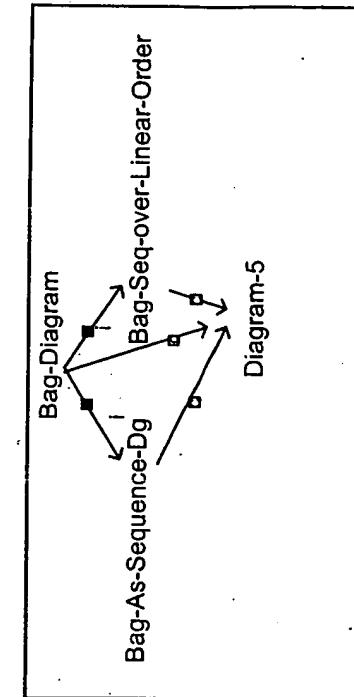


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Example user interface showing a hereditary diagram
Fig. 8(a)

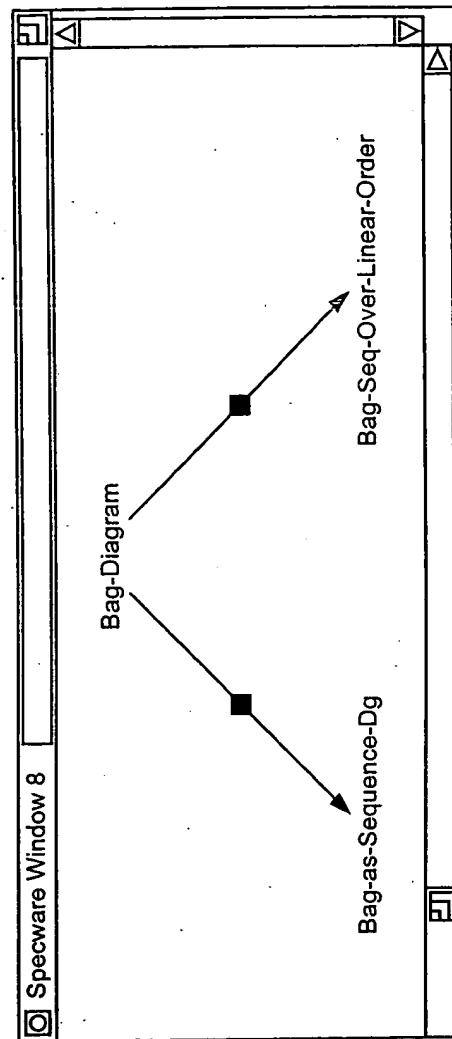
Example user interface showing a hereditary diagram (interface for user to indicate "find colimit" operation)
Fig. 8(b)



Example user interface showing a hereditary diagram after the user indicates a "find colimit" operation for the hereditary diagram and the colimit operation is performed
Fig. 8(c)



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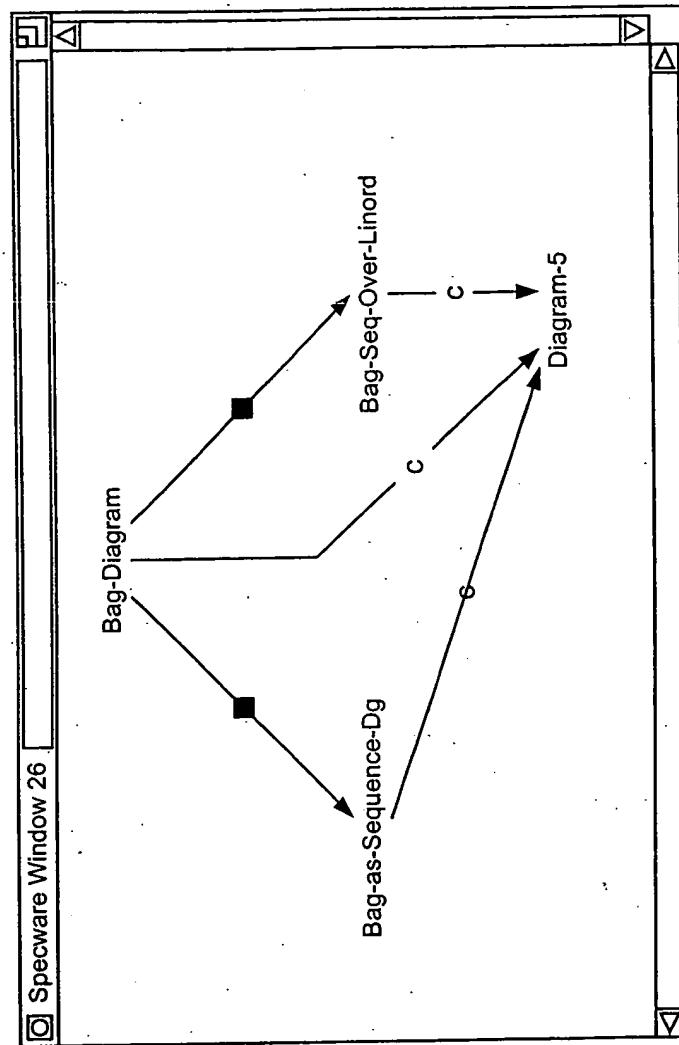


Heredity diagram

Fig. 9(a)



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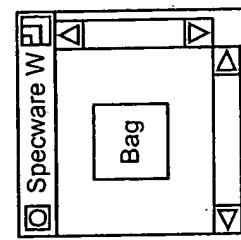


Hereditary diagram, including colimit

Fig. 9(b)



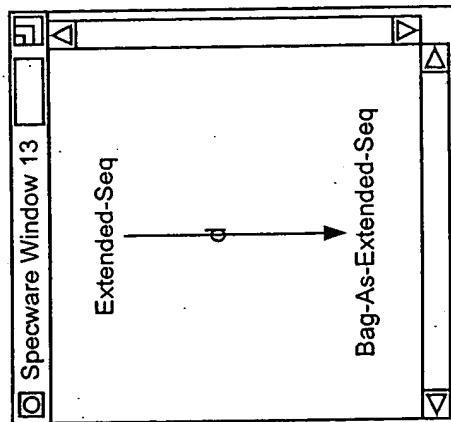
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Bag diagram
(obtained by expanding node
"Bag-Diagram
in Hereditary diagram")

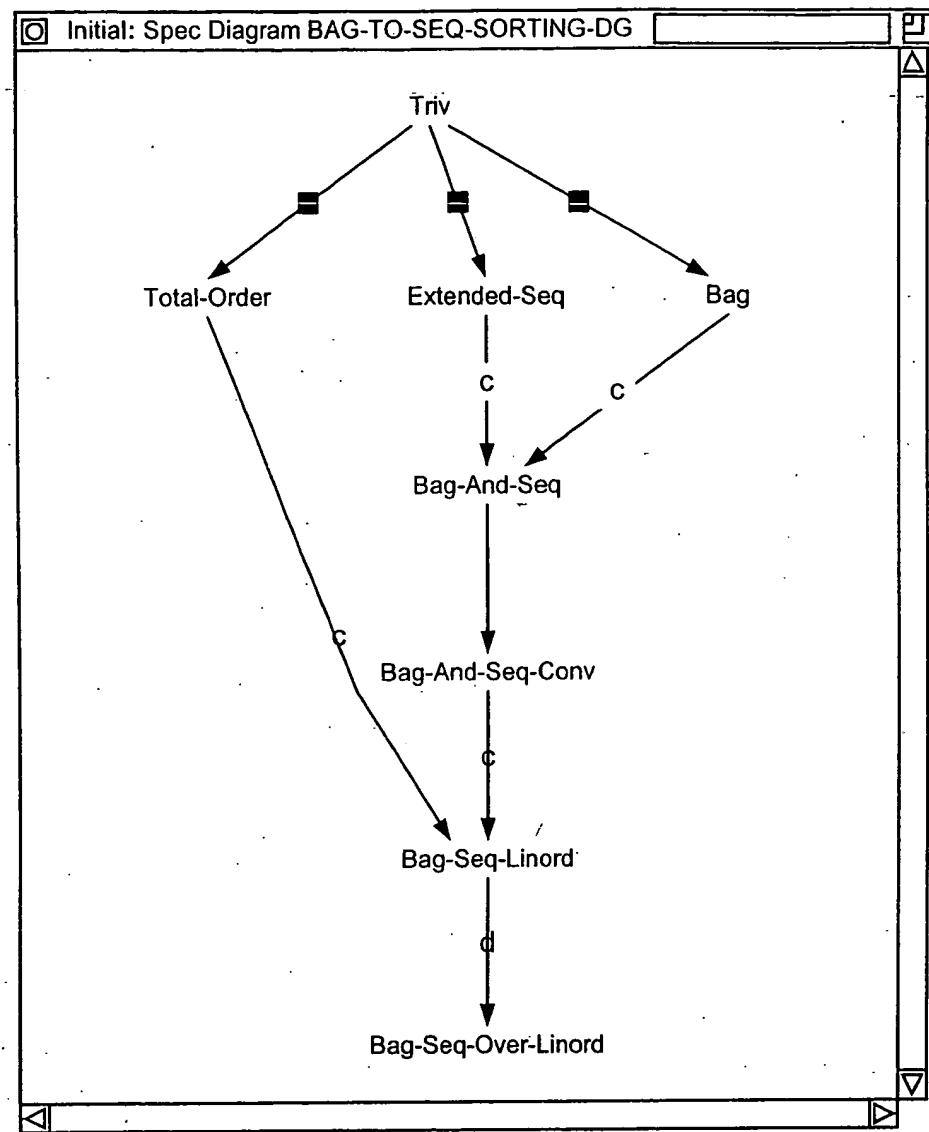
Fig. 9(c)

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Bag-as-Sequence diagram
(obtained by expanding node
Bag-as-Sequence-diagram
in Hereditary diagram)

Fig. 9(d)

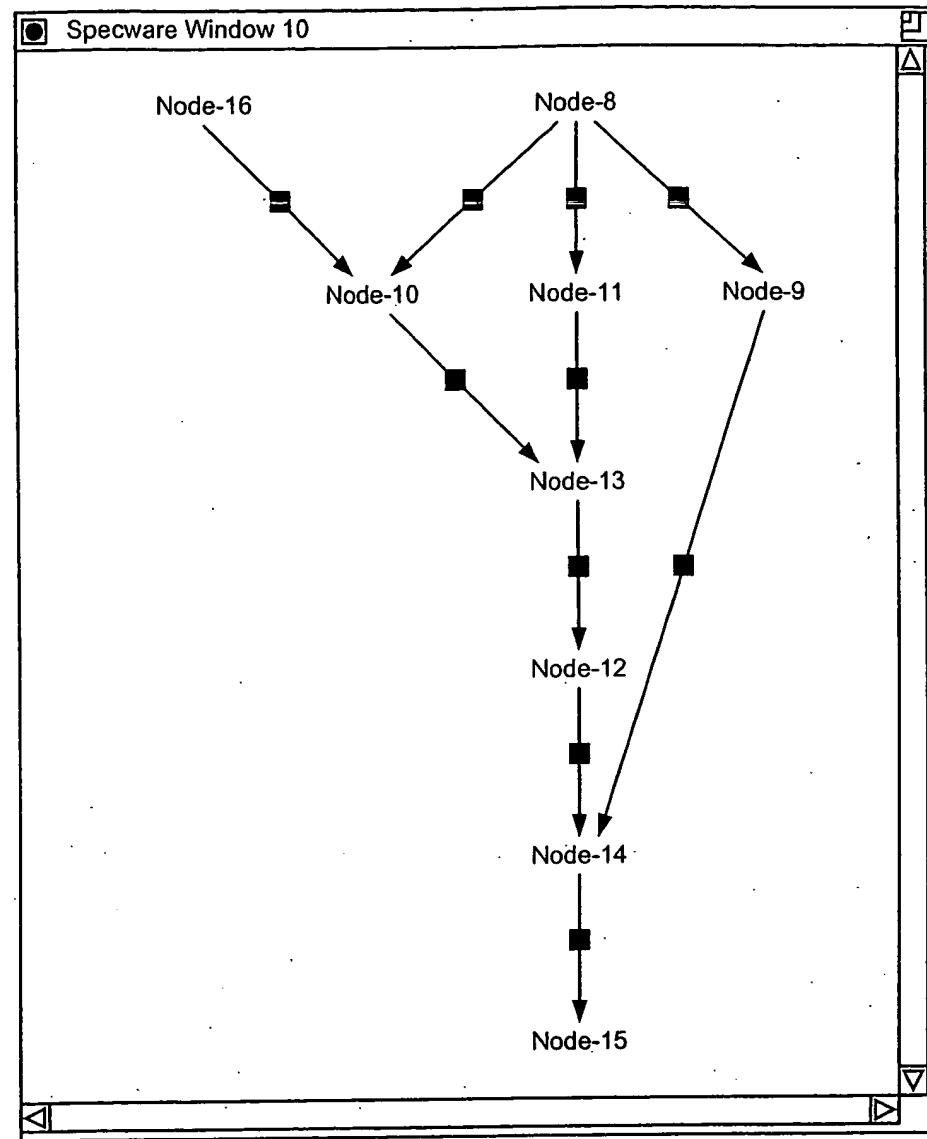


Bag-Seq-over-Linear-Order diagram
(obtained by expanding node
Bag-Seq-over-Linear-Order-
diagram
in Hereditary diagram)

Fig. 9(e)

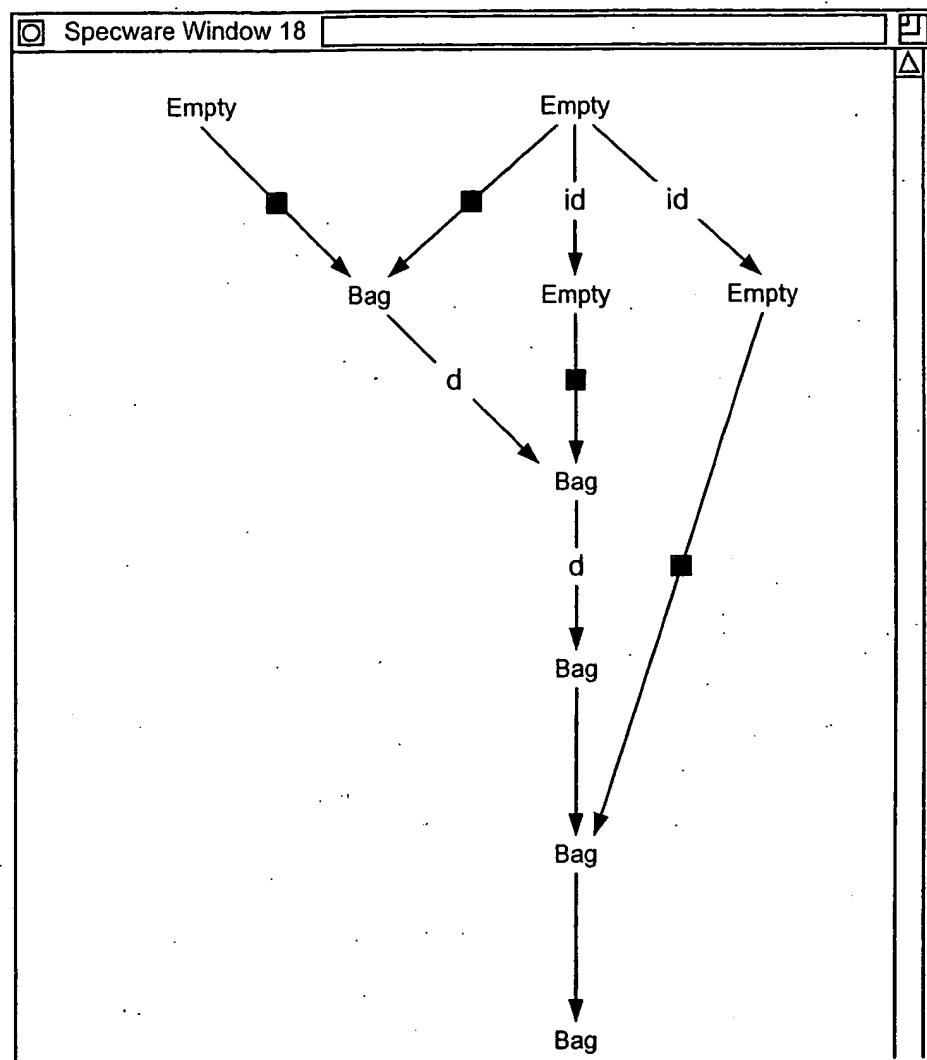


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Shape of colimit

Fig. 9(f)

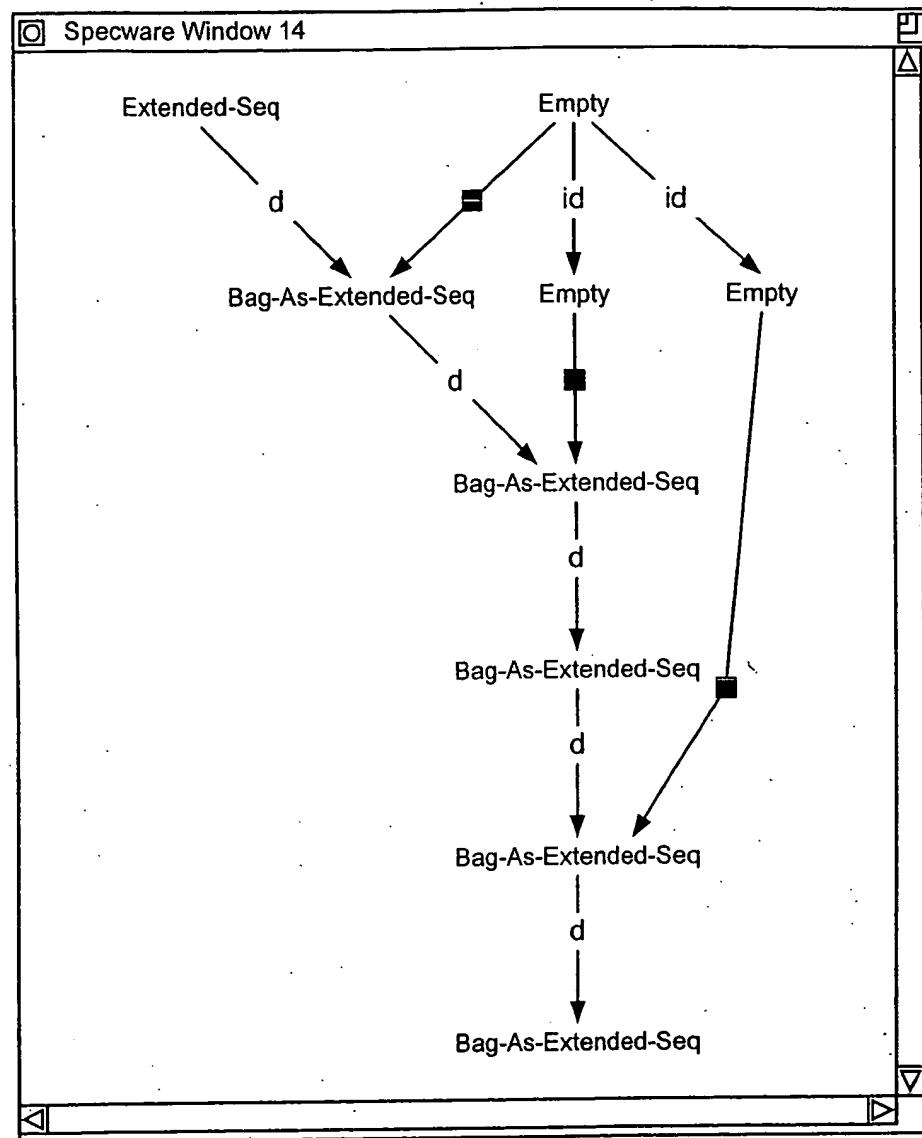


Extended Bag diagram

Fig. 9(g)



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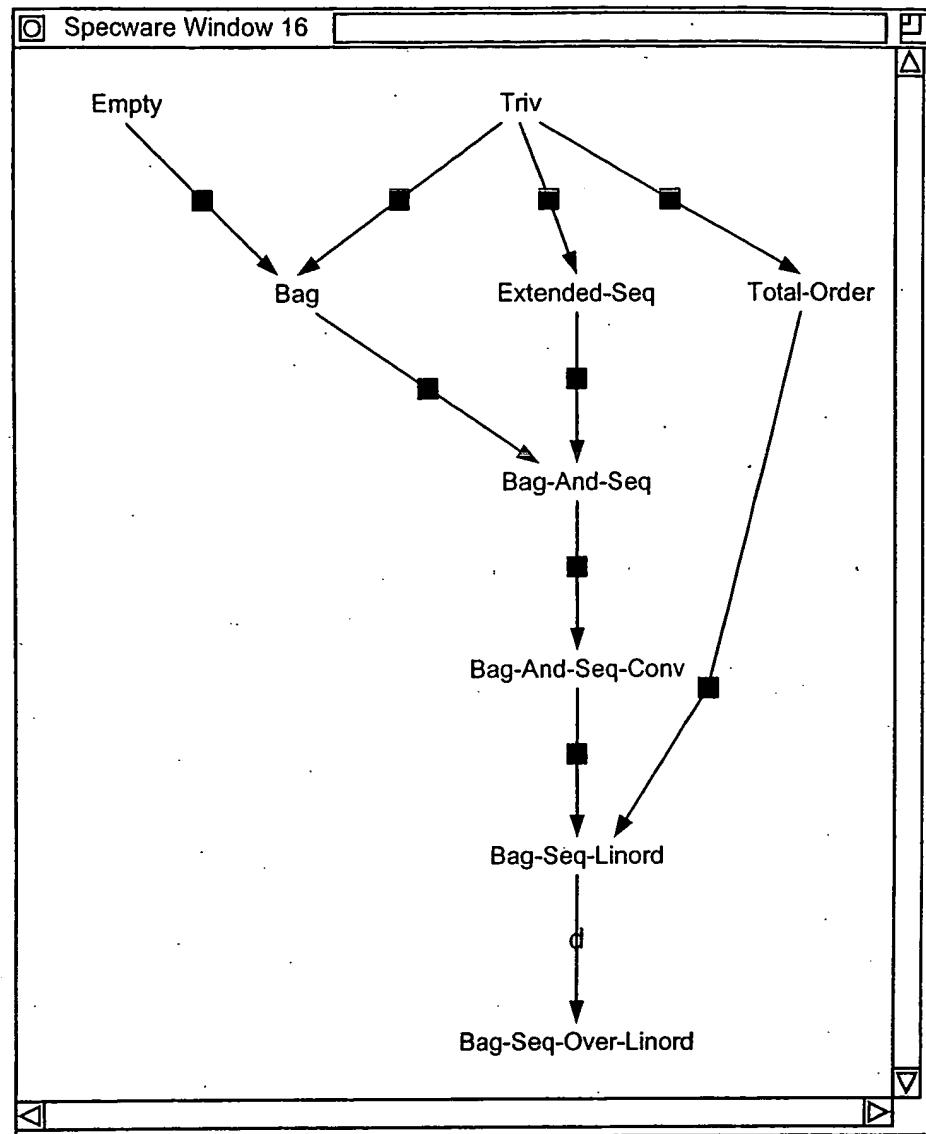


Extended Bag-as-Sequence diagram

Fig. 9(h)



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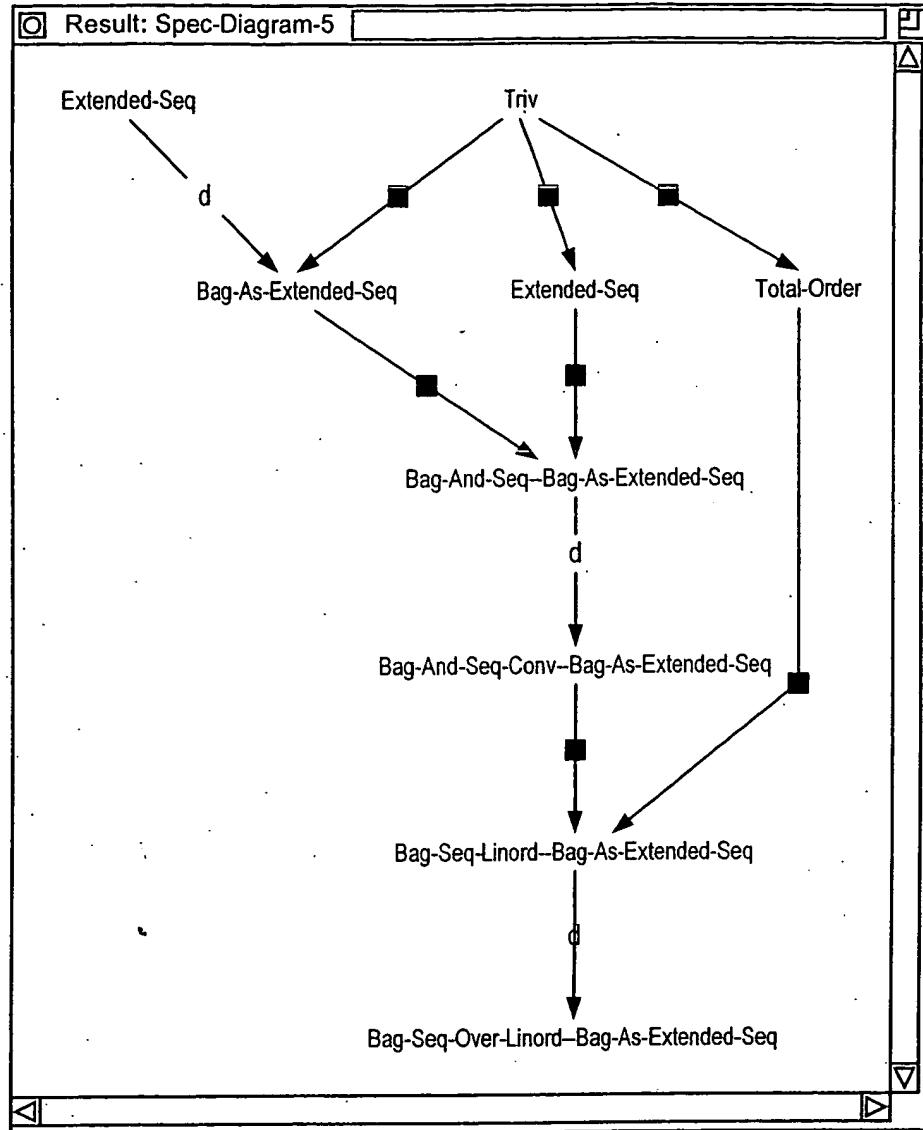


Extended Bag-Seq-over-Linear-Order diagram

Fig. 9(i)

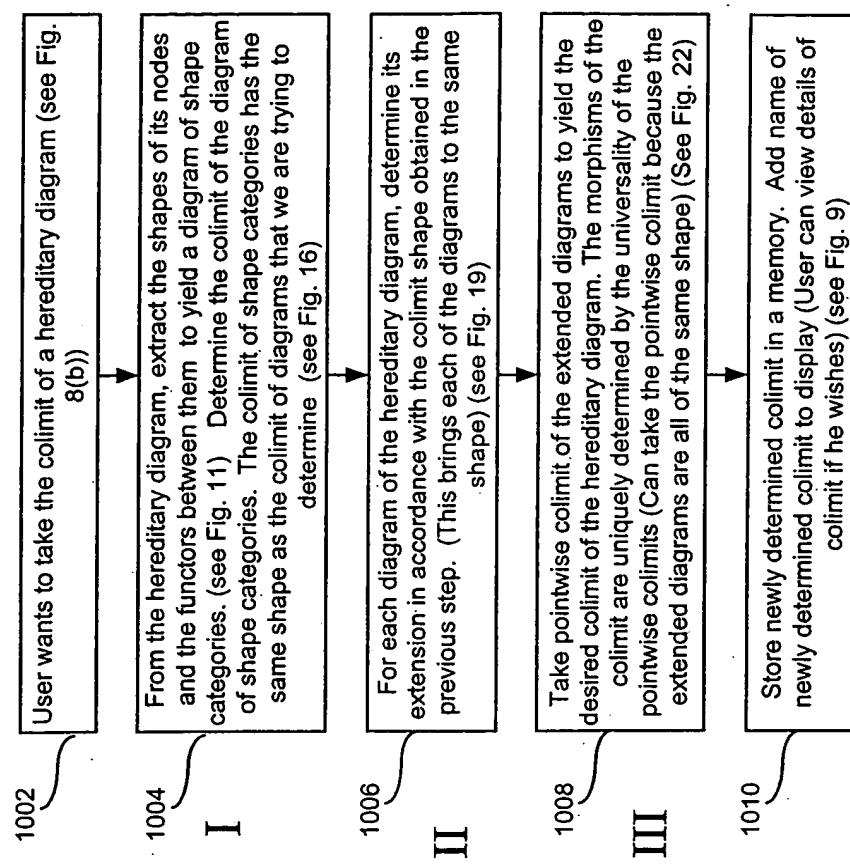


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Colimit of Hereditary diagrams
(final result)

Fig. 9(j)

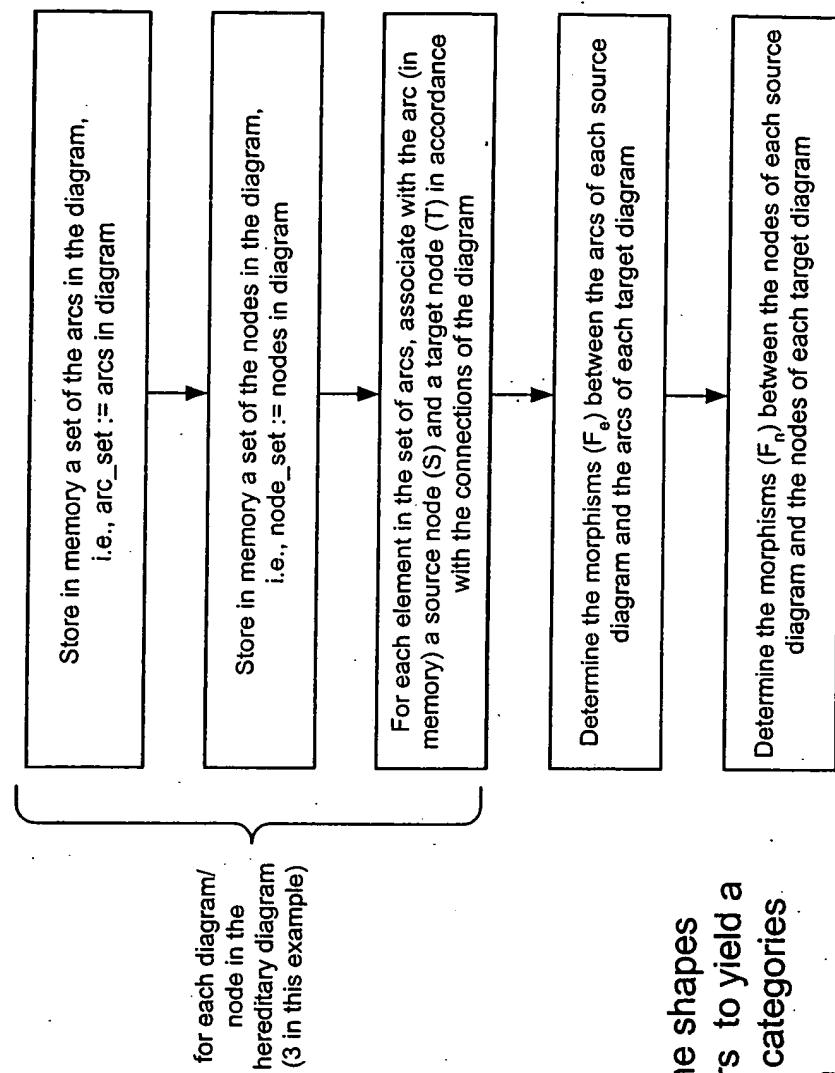


Finding a Colimit of
Hereditary Diagrams

Fig. 10



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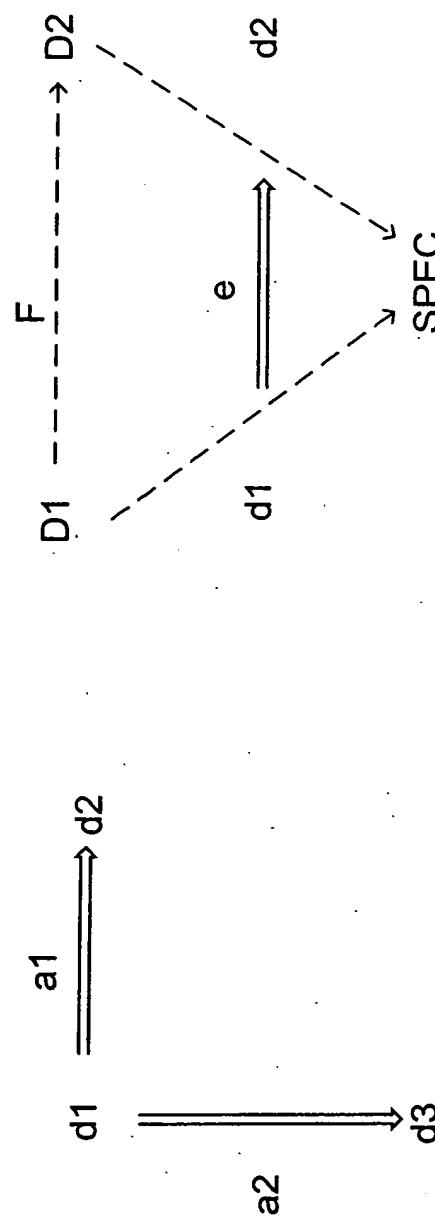


PART I: Extract the shapes and shape functors to yield a diagram of shape categories.

Fig. 11



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A Hereditary Diagram: Three Diagrams and Two Arcs.

Each arc a_1 and a_2 represents a shape morphism having 1) a shape functor (such as F) and 2) a natural shape transformation (such as $e: d_1 \rightarrow d_2$)

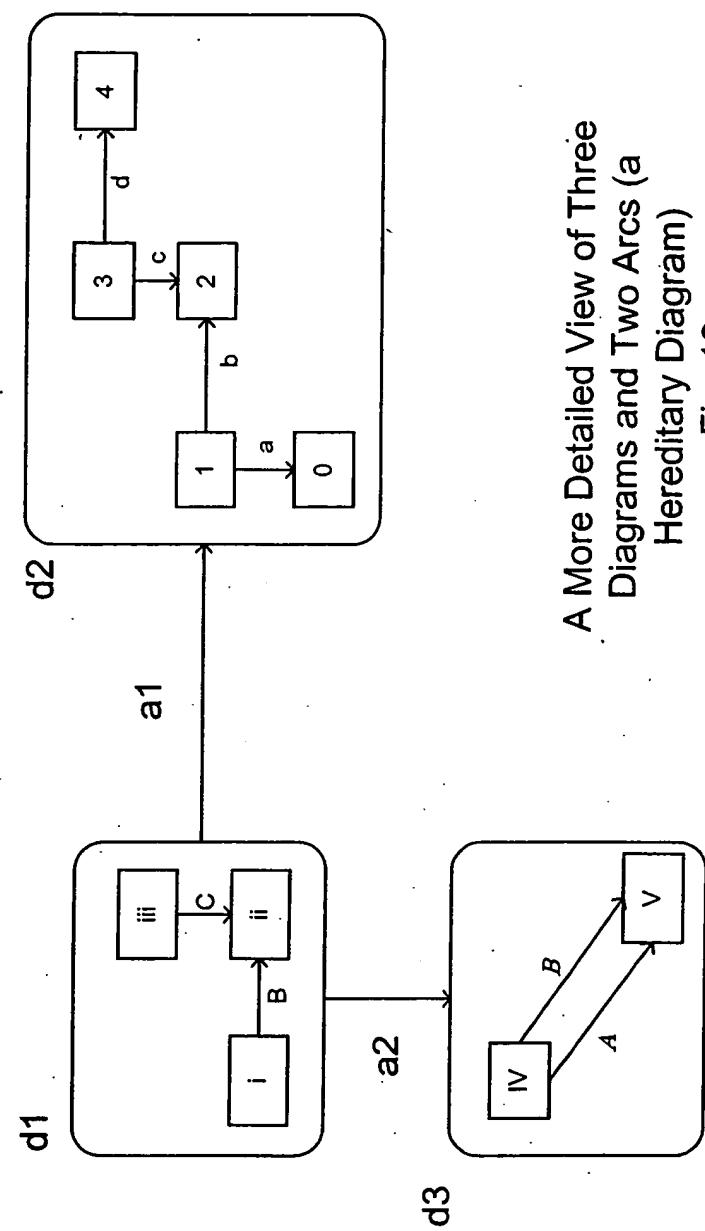
Fig. 12(a)

A Shape Morphism

where d_1 and d_2 are diagrams,
 F is a shape functor,
 e is a natural transformation from d_1 to
 $(d_2 \text{ composed with } F)$
 D_1 and D_2 are shape categories of
diagrams, and $SPEC$ is the category
Spec

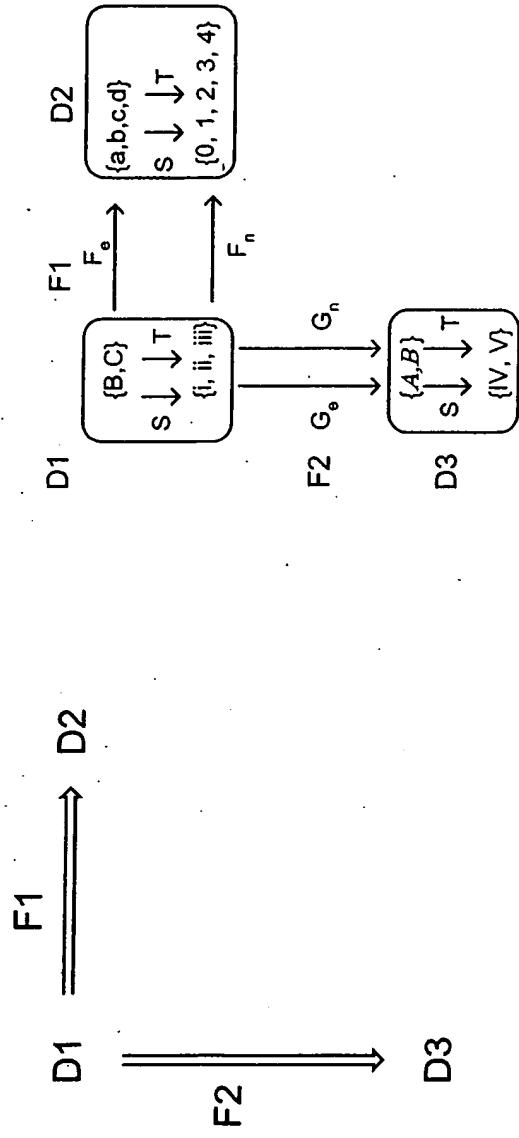
Fig. 12(b)

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Extract the Shapes and
 Shape Functors
 (D1 is shape of diagram
 d1, F1 is shape functor)
 Fig. 14

More Detailed View of Extracting the
 Shapes and Shape Functors
 (continued on Figs. 15(b)-15(d))
 Fig. 15(a)

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Arcs: B \rightarrow A
C \rightarrow B
Nodes:
i \rightarrow 1
ii \rightarrow 2
iii \rightarrow 3

Arcs: B \rightarrow b
C \rightarrow c
Nodes:
i \rightarrow 1
ii \rightarrow 2
iii \rightarrow 3

Mapping for F1
Fig. 15(b)

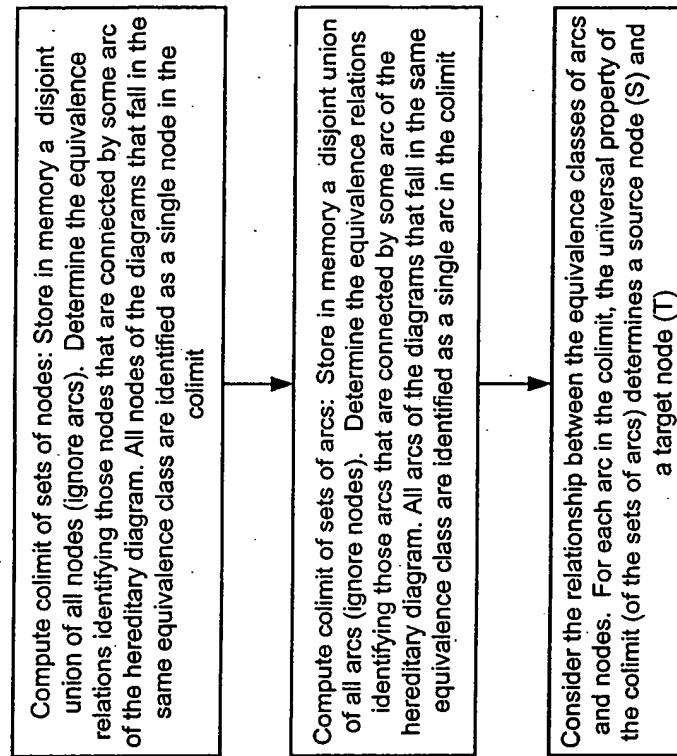
Arcs: B \rightarrow A
C \rightarrow B
Nodes:
i \rightarrow IV
ii \rightarrow V
iii \rightarrow IV

Arcs: B \rightarrow A
C \rightarrow B
Nodes:
i \rightarrow 1
ii \rightarrow V
iii \rightarrow IV

Mapping for F2
Fig. 15(c)

Source (S) and
Target (T)
Functions for
Hereditary
Diagrams
Fig. 15(d)

Arc	B	C	a	b	c	d	A	B
Source	i	iii	1	1	3	3	IV	IV
Target	ii	ii	0	2	2	4	V	V

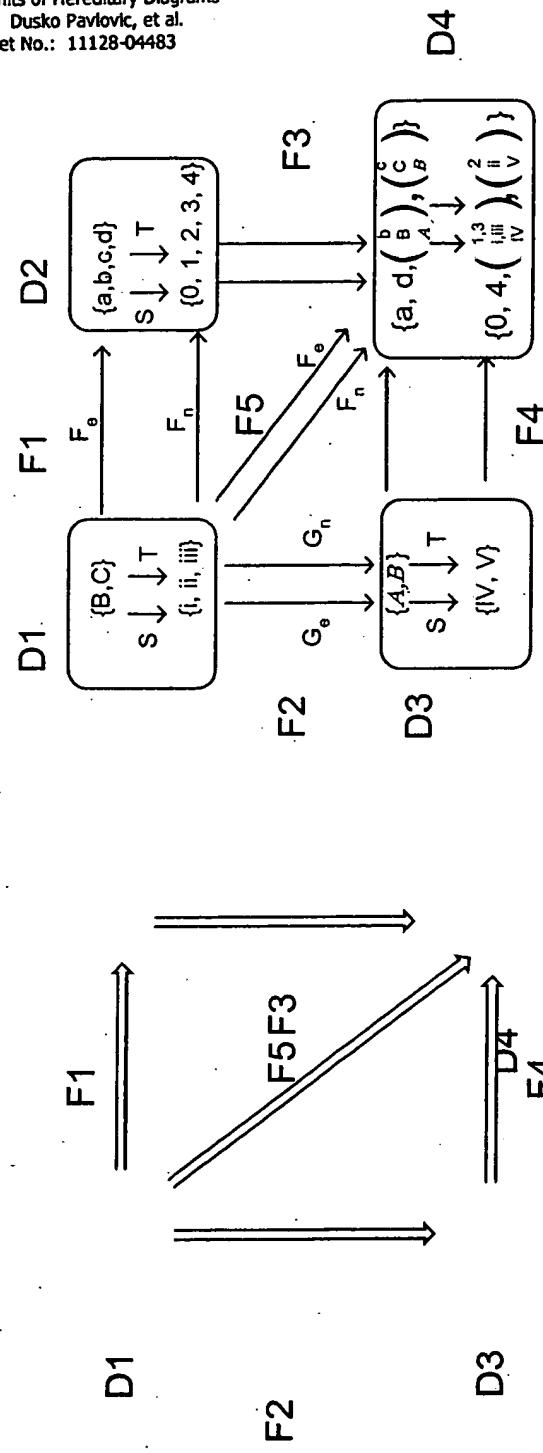


PART I: Determine the colimit of the diagram of shape categories.

Fig. 16



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More Detailed View of Taking the Colimit
 (See Figs 18(b)-(f))

Fig. 18(a)

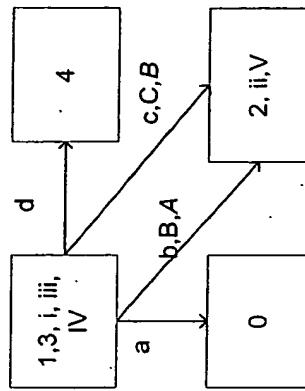
More Detailed View of
 Taking the Colimit
 Fig. 17



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	Arc	a	d	b	c	Source	Targ	Function	Shape C	Fig.
						1.3	1.3	1.3	i,iii	v
						i,iii	iv	iv	iv	v
						source	target	function	shape c	figure
						target	target	target	target	target

Source (S) and
Target (T)
Functions for
Shape Colimit D4
Fig. 18(b)



The Colimit D4 of the Shape Diagrams
Fig. 18(c)

Fig. 18(c)

Arcs: B → b,B,A
C → c,C,B

Nodes: i -> 1,3, i, iii, iv
ii -> 2, ii, v
iii -> 1,3, i, iii, iv

Mapping for F4
Fig. 18(e)

Mapping for F3
Fig. 18(d)

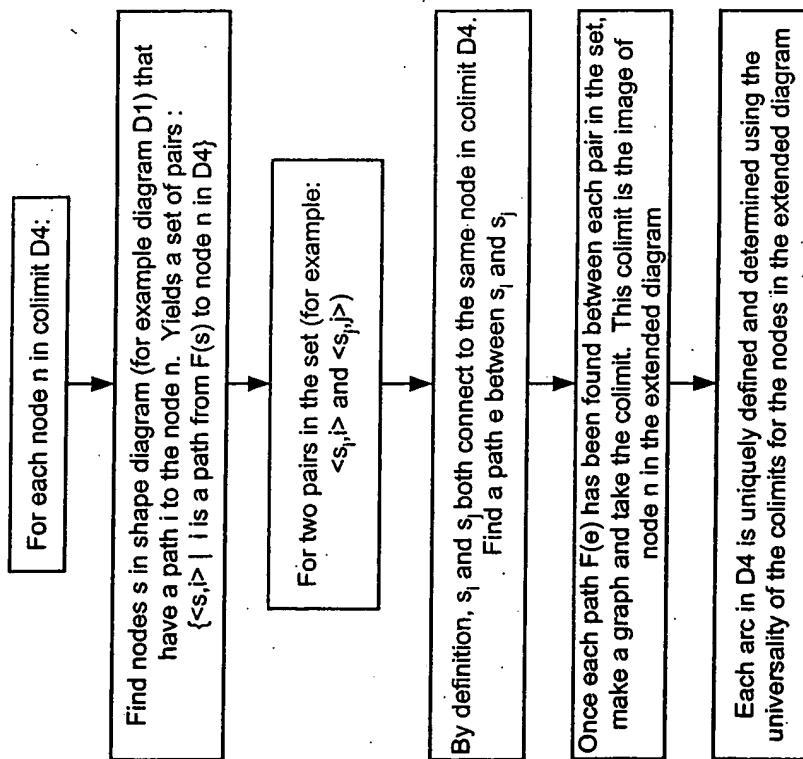
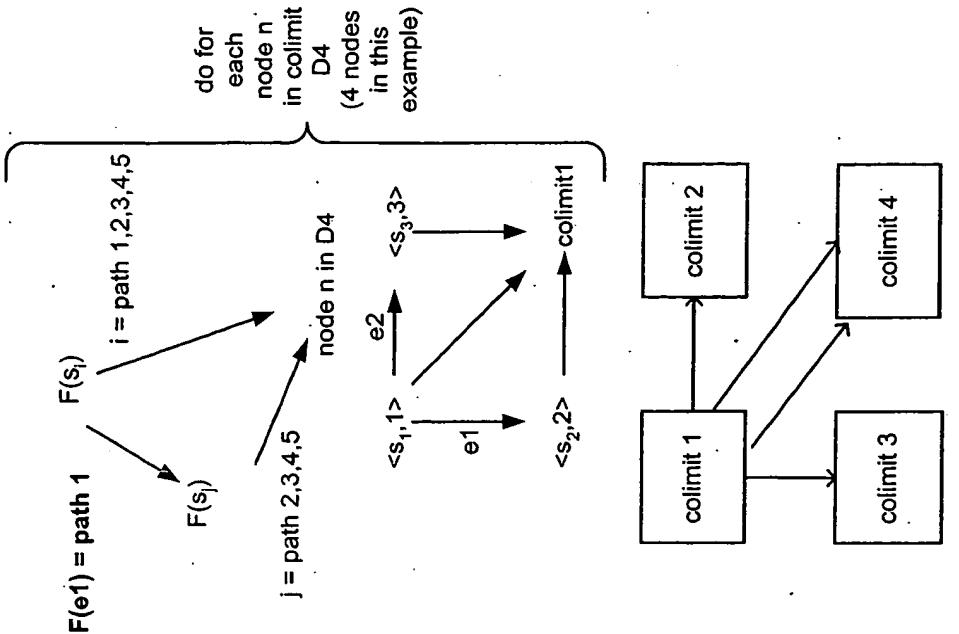
Arcs:	$a \rightarrow a$	$0 \rightarrow 0$
	$d \rightarrow d$	$1 \rightarrow 1, 3, i, iii, IV$
	$b \rightarrow b, B, A$	$2 \rightarrow 2, ii, V$
	$c \rightarrow c, C, B$	$3 \rightarrow 1, 3, i, iii, IV$
		$4 \rightarrow 4$

Nodes: 0 -> 0
 1 -> 1,3, i,iii, IV
 2 -> 2, ii,V
 3 -> 1,3, i,iii, IV
 4 -> 4

Nodes: 0 -> 0
 1 -> 1,3, i,iii, IV
 2 -> 2, ii,V
 3 -> 1,3, i,iii, IV
 4 -> 4



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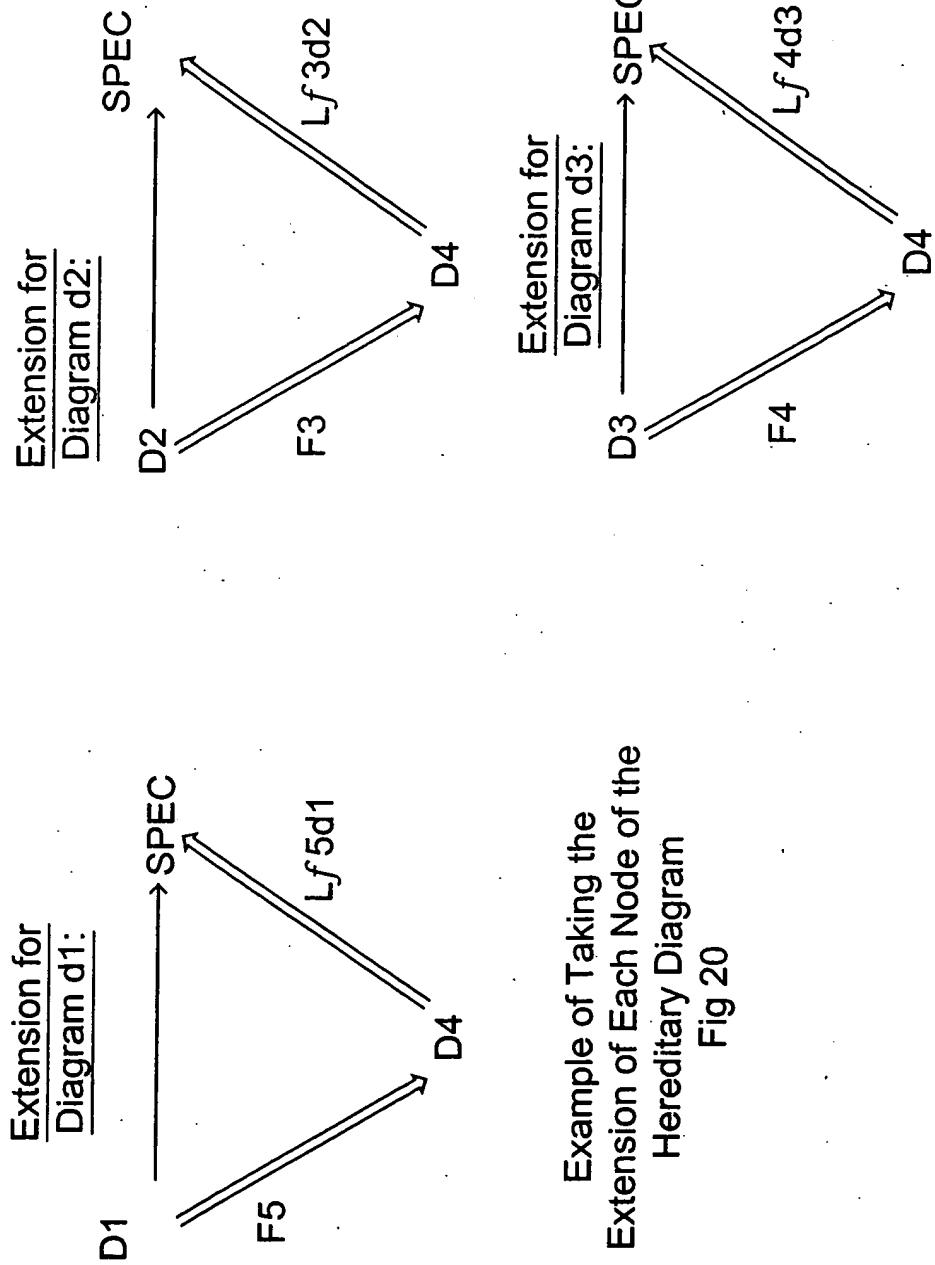


PART II: Extending one Diagram (repeat to extend each diagram) Fig. 19

extended diagram from example

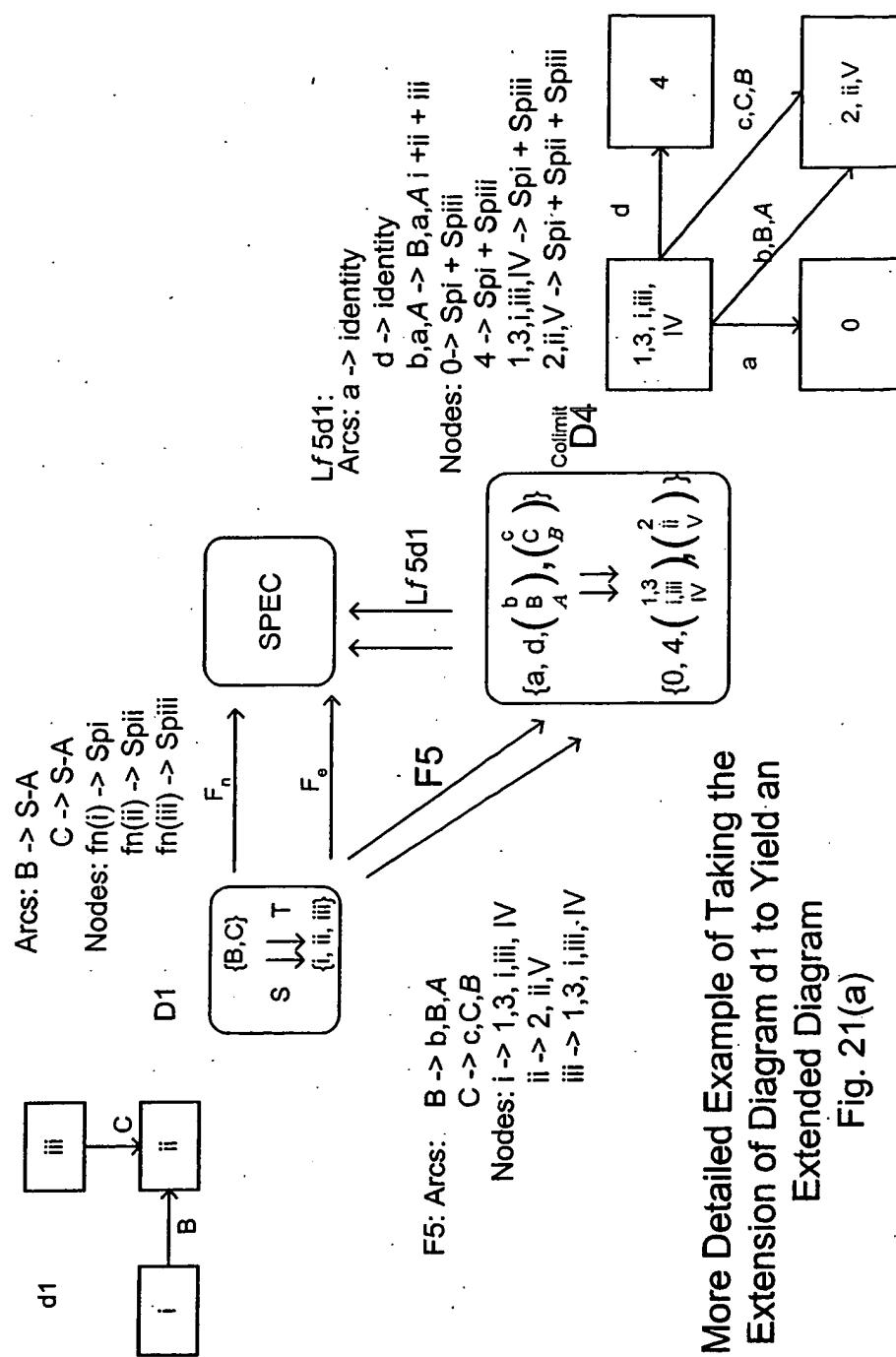


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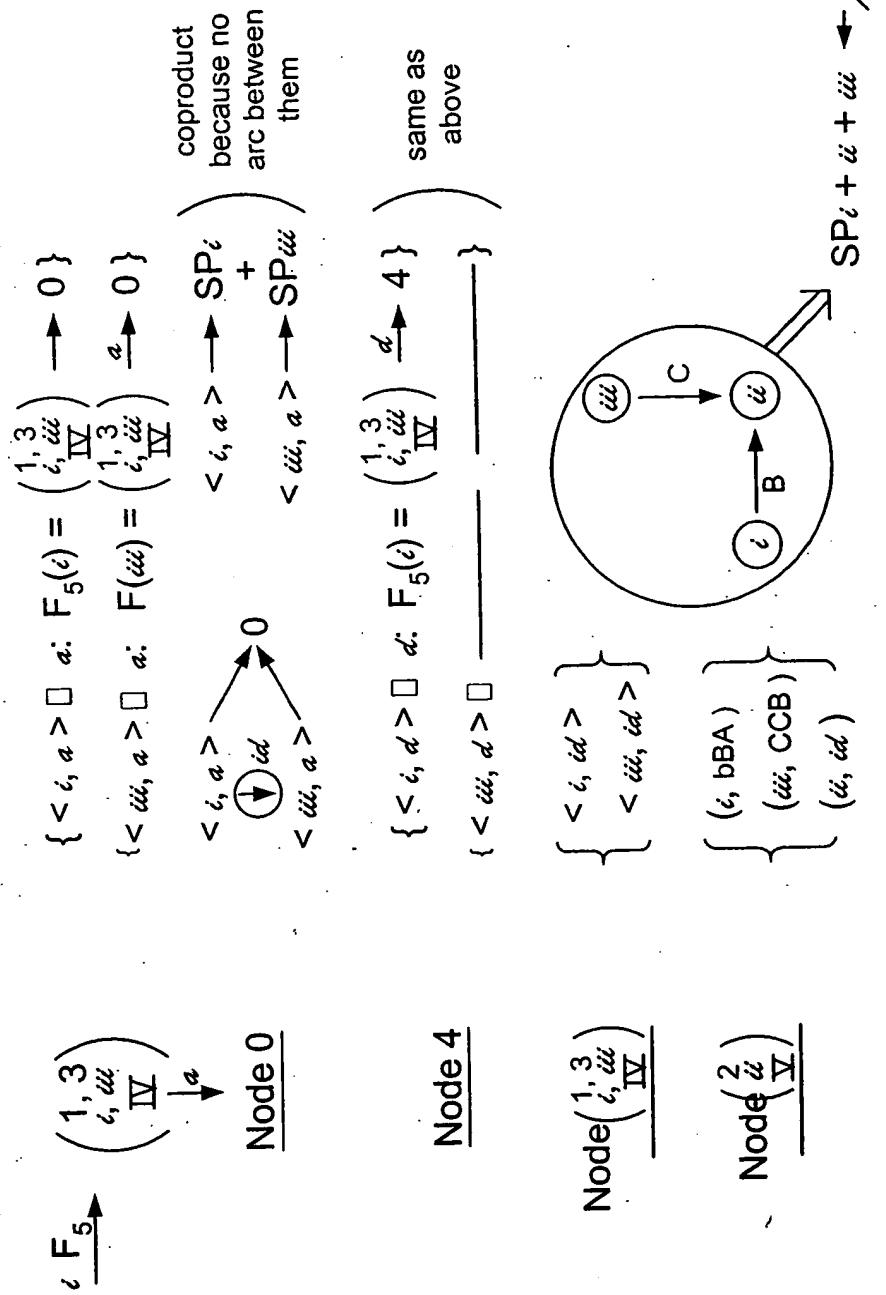


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Example of Diagram Extension.

Fig. 21(b)



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Fig 15

Diagram showing a circle with center a and radius $c/\sqrt{2}$. A point 0 is on the circumference. The angle at the center between the radius to 0 and the horizontal diameter is labeled as $1, 3$ and IV .

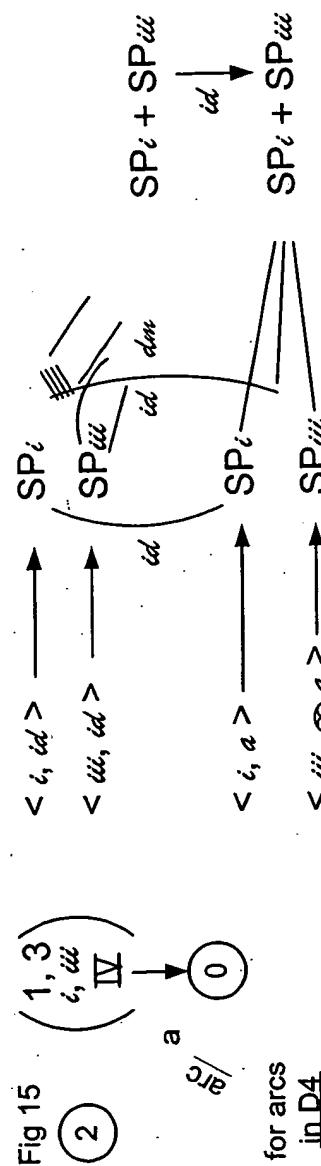
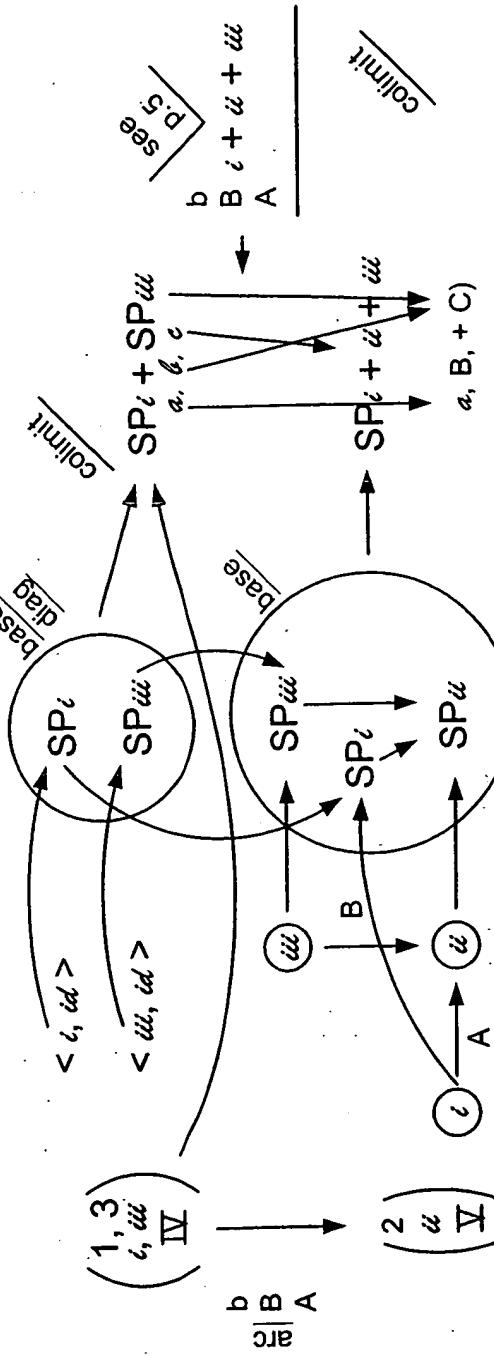
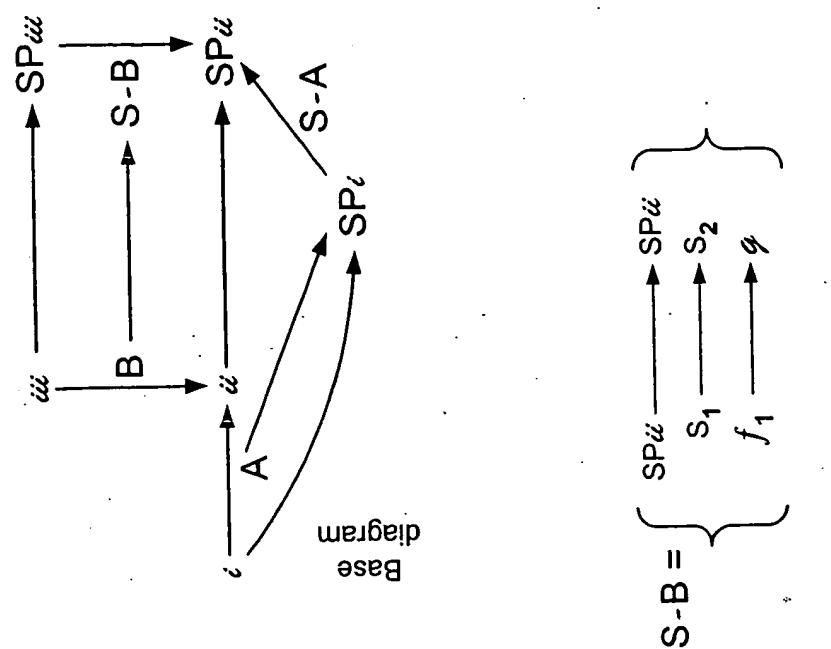


Fig. 21(c) Example of Diagram Extension (cont)





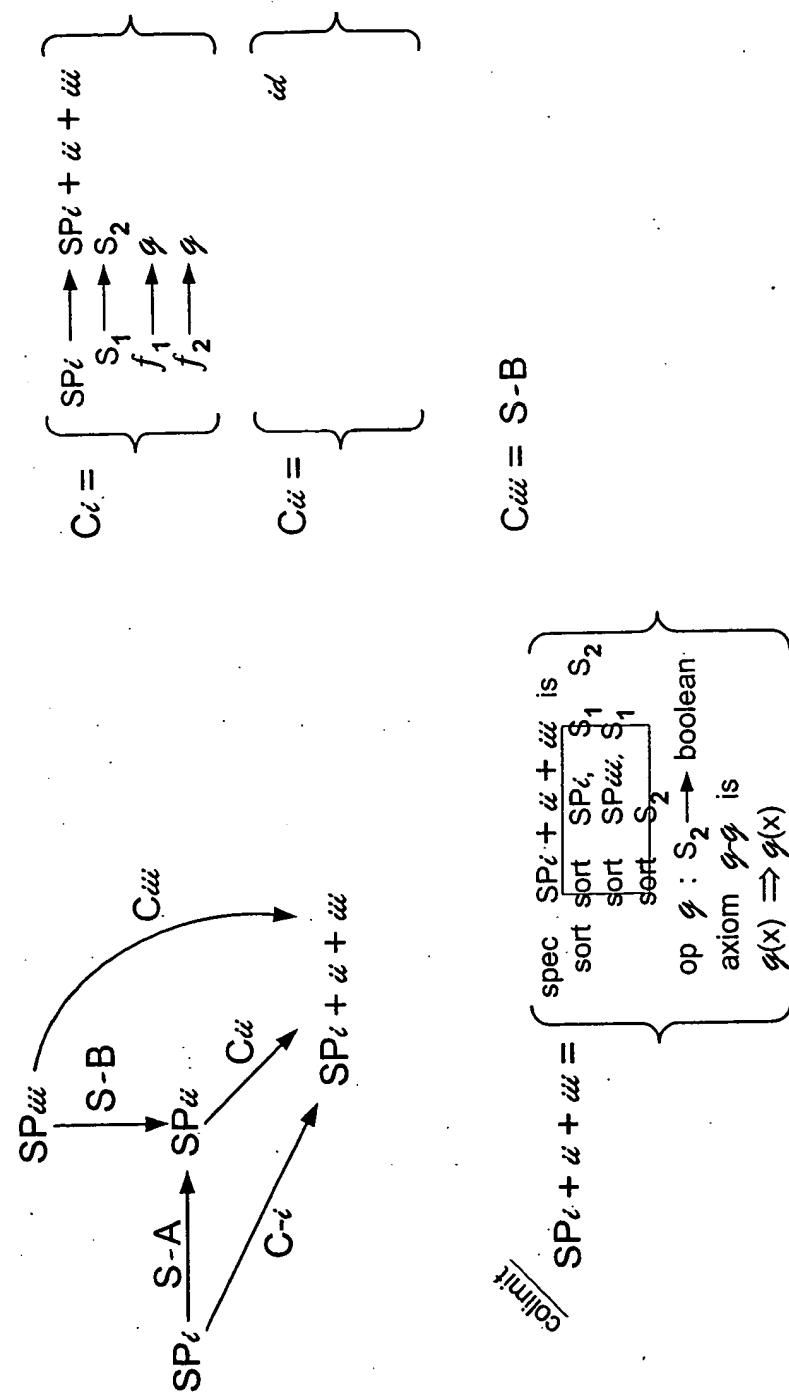
$$\begin{aligned}
 SP_L &= \left\{ \begin{array}{l} \text{spec } SP_L \text{ is} \\ \text{sort } S_1 \\ \text{op } f_1 : S_1 \rightarrow \text{boolean} \\ \text{op } f_2 : S_1 \rightarrow \text{boolean} \\ \text{axiom } f_1 \Rightarrow f_2 = \text{is} \\ f_1(x) \Rightarrow f_2(x) \end{array} \right\} \\
 SP_{UU} &= \left\{ \begin{array}{l} \text{spec } SP_{UU} \text{ is} \\ \text{sort } S_2 \\ \text{op } g : S_2 \rightarrow \text{boolean} \end{array} \right\} \\
 SP_{UU} &= \left\{ \begin{array}{l} \text{spec } SP_{UU} \text{ is} \\ \text{sort } S_1 \\ \text{op } d_1 : S_1 \rightarrow \text{boolean} \end{array} \right\} \\
 S-A &= \left\{ \begin{array}{l} SP_L \longrightarrow SP_{UU} \\ S_1 \longrightarrow S_2 \\ f_1 \longrightarrow g \\ f_2 \longrightarrow g \end{array} \right\}
 \end{aligned}$$

Fig. 21(d)

Example of Diagram Extension (cont)



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Example of Diagram Extension (cont)

Fig. 21(e)



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$$SP_i + SP_{iii} = \left\{ \begin{array}{l} \text{sort } SP_i \cdot S_1 \\ \text{sort } SP_{iii} \cdot S_1 \\ \text{op } SP_i \cdot f_1 : SP_i \cdot S_1 \rightarrow \text{boolean} \\ \text{op } SP_{iii} \cdot f_1 : SP_{iii} \cdot S_1 \rightarrow \text{boolean} \\ \text{op } SP_i \cdot f_2 : SP_i \cdot S_1 \rightarrow \text{boolean} \end{array} \right\}$$

↓
 b
 B $i + ii + iii$
 A

$$SP_i + ii + iii = \left\{ \text{See Fig. 21(e)} \right\}$$

$$b
 B $i + ii + iii$
 A = \left\{ \begin{array}{l} SP_i \cdot S_1 \longrightarrow S_2 \\ SP_{iii} \cdot S_1 \longrightarrow S_2 \\ SP_i \cdot f_1 \longrightarrow g \\ SP_{iii} \cdot f_1 \longrightarrow g \\ SP_{iii} \cdot f_2 \longrightarrow g \end{array} \right\}$$

Example of Diagram Extension (cont)

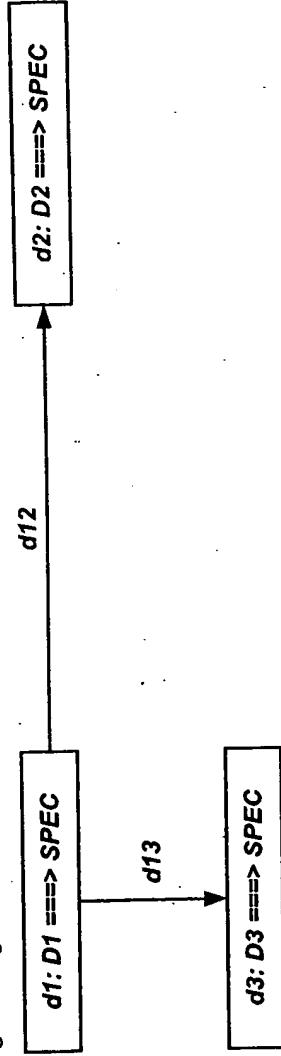
Fig. 21(f)



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After finishing the extension for each diagrams, let us use the following example:

Original diagrams:



Its underlying shape categories, shape functors and the colimit are:

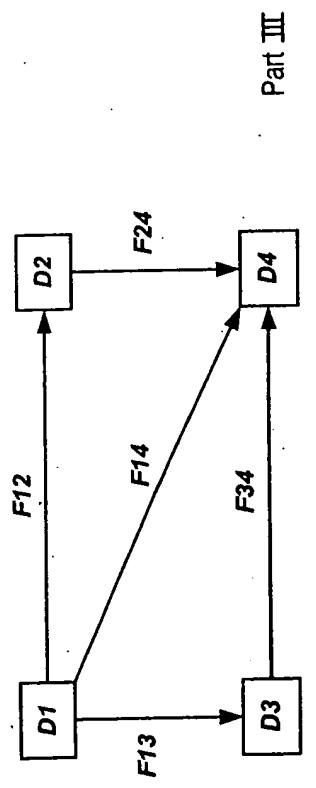
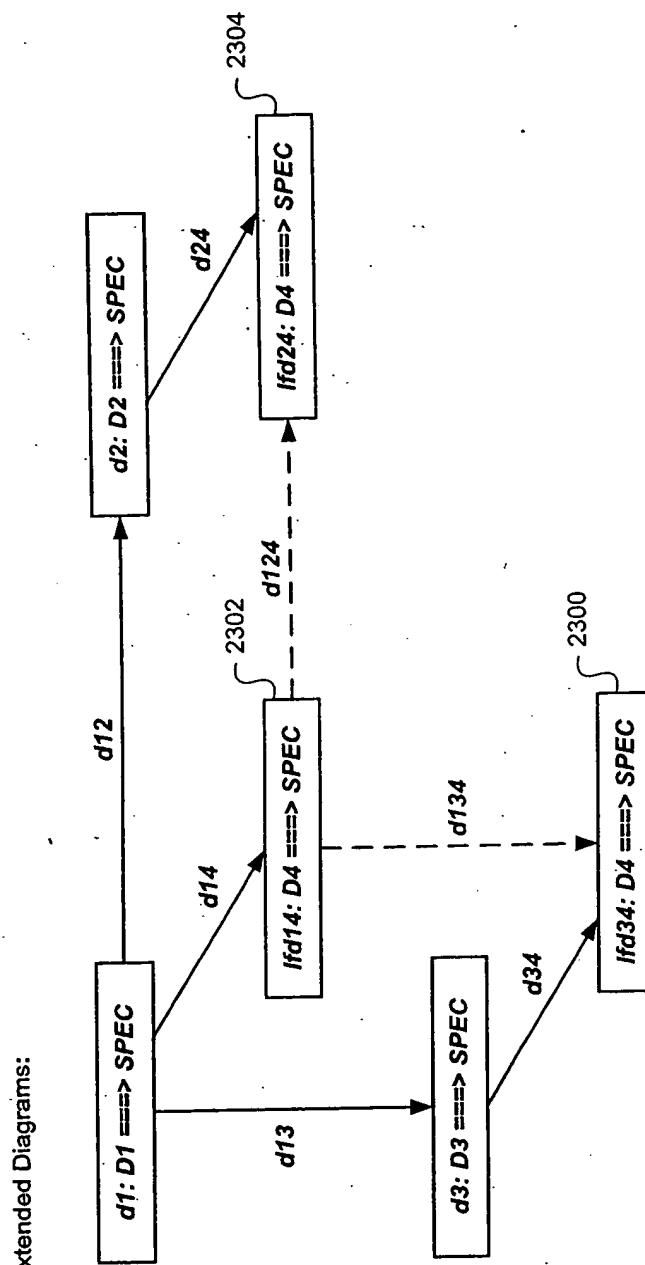


Fig. 22
Part III

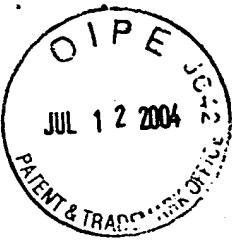


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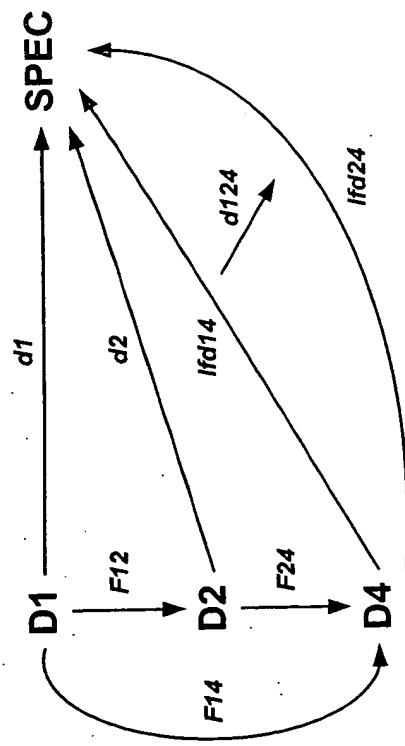


The last algorithm step we are missing for constructing the diagram colimits is the diagram morphisms between extended diagrams. For example, the diagram morphism $d1d4$ and $d134$ (dotted lined arrows in above figure) are the ones needed. Suppose $d1d4$ and $d134$ are two extensions of $d1$ and $d2$, given the colimit of the shape categories as $D4$. We would have the following picture.

Fig. 23



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A morphism between $lfd14$ and $lfd24$ is a natural transformation which maps each node of $D4$ to an arrow in $SPEC$. We ?? ???

The universal construction of witness arrows.

For any node n_i in $D4$, we have $F14(n_i) = F12 \circ F24(n_i)$. Let $Sp1ni$ and $Sp2ni$ two shape categories used for constructing a mapping for n_i in its extension of $d1$ and $d2$, respectively, then we can have a shape function between $Sp1ni$ and $Sp2ni$ (inclusion, basically). That induces a diagram morphism between the base diagrams for the target of n_i in $lfd14$ and $lfd24$, respectively. By imposing that diagram morphism and cocone morphism, we can get an unique arrow between $lfd14(n_i)$ and $lfd24(n_i)$. Repeating this process we construct a natural transformation between $lfd14$ and $lfd24$. Similarly, we can do this for any two extended diagrams.

The following flowchart is the algorithm for constructing a diagram morphism between two extended diagrams.

Fig. 24

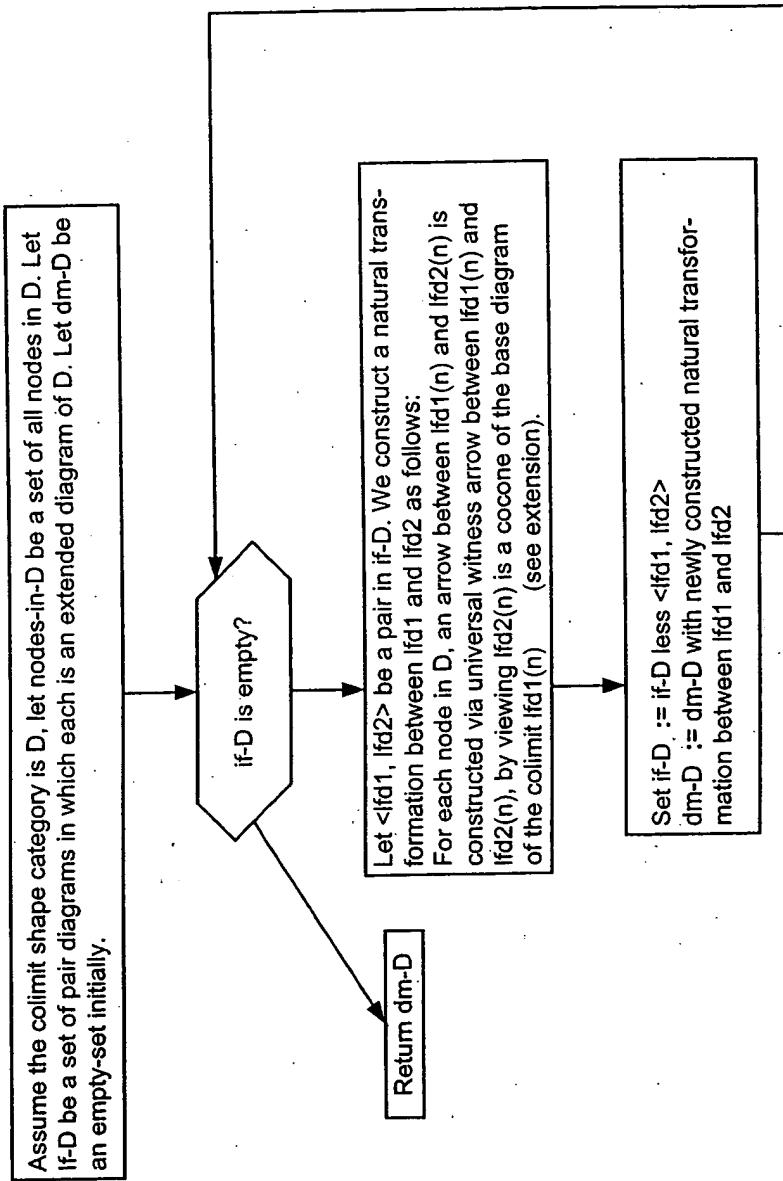
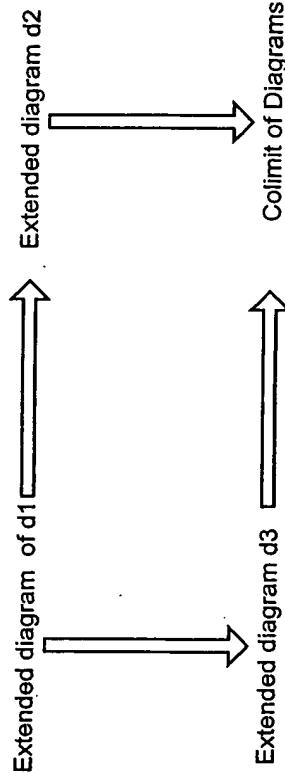


Fig. 25



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The final step is to complete the colimit of the extended diagrams. The colimit is determined by computing the pointwise colimits over corresponding nodes in the extended diagrams. The morphisms are computed uniquely using universality of the pointwise colimits.



Taking Pointwise Colimit of Extended Diagrams
(Can be done, since extended diagrams are all the same shape)
Fig. 26



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Diagram	Hereditary Diagram	Arc	Shape Morphism Graph
	Arc; source and target diagrams	Shape functor (F_e)	Diagram Category Pair
	...	Natural Transformation (F_n)	Arc
	Arc; source and target diagrams		
	...		
	Arc; source and target diagrams		
	Total number of Arcs		

Examples of Data Structures used in the Example Implementation
Fig. 27

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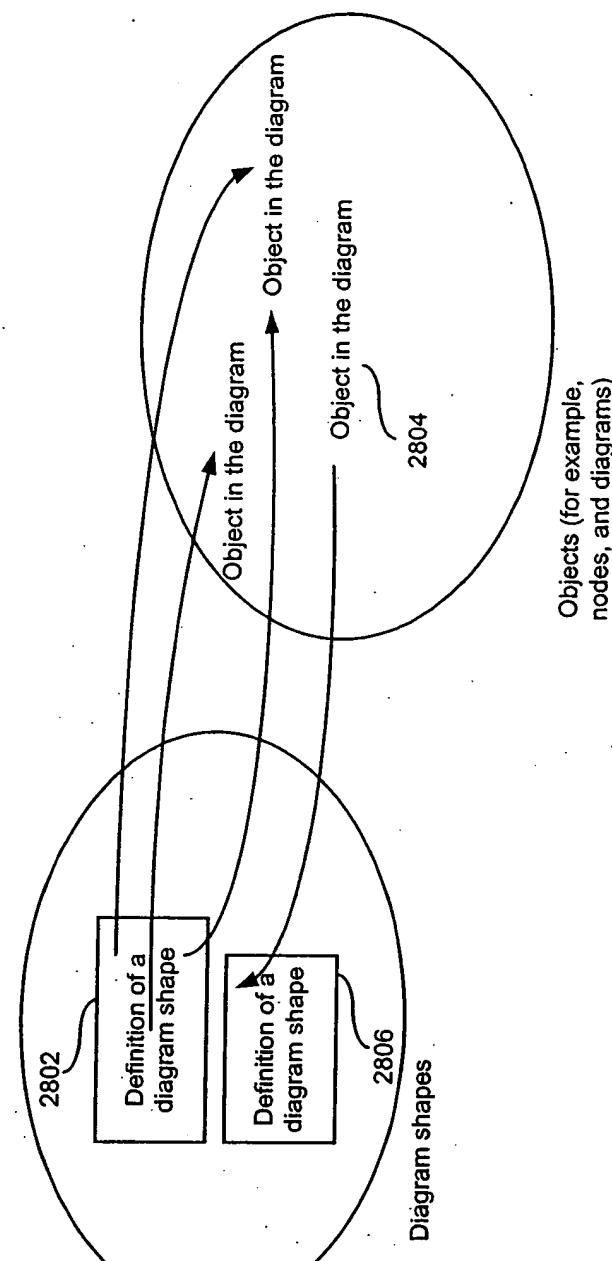


Fig. 28



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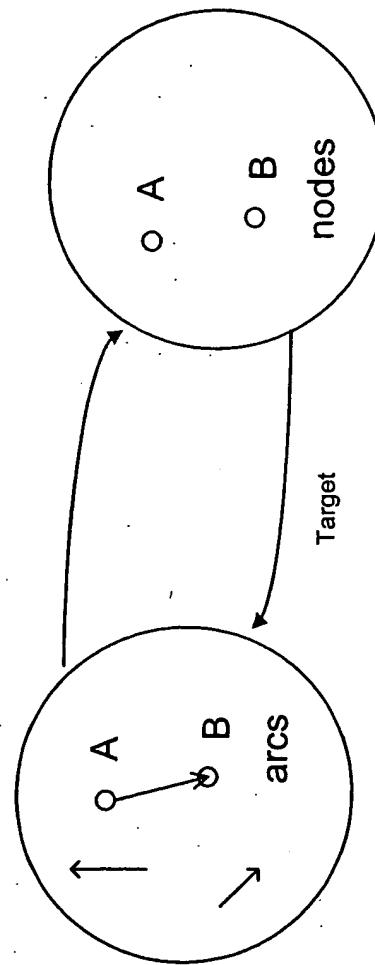


Fig. 29